# Chapter 26
# Current and Resistance

**In this chapter we will introduce the following new concepts:**

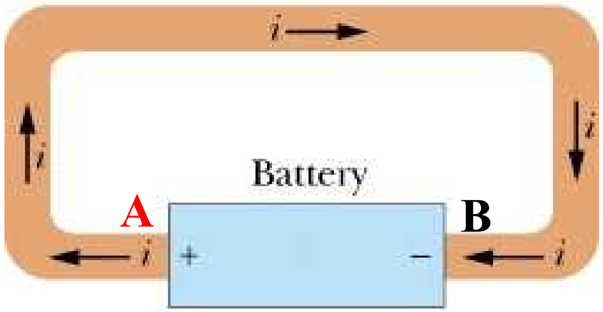
-Electric current ( symbol $i$ )
-Electric current density vector (symbol $\vec{J}$ )
-Drift speed (symbol $v_d$ )
-Resistance (symbol $R$ ) and resistivity (symbol $\rho$ ) of a conductor
-Ohmic and non-Ohmic conductors

**We will also cover the following topics:**

-Ohm's law
-Power in electric circuits

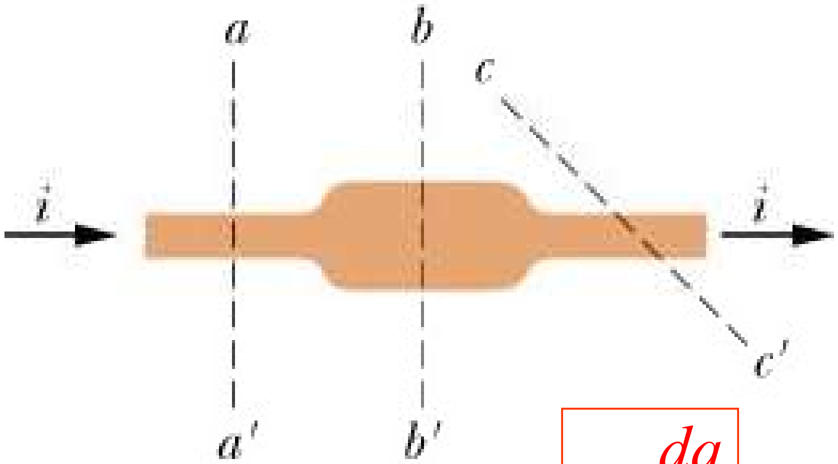
## Electric Current

(a)




(b)

Consider the conductor shown in fig. *a*.
All the points inside the conductor and on its surface are at the same potential.
The free electrons inside the conductor move in random directions and thus there is no net charge transport.
We now make a break in the conductor and insert a battery as shown in fig. *b*.
Points $A$ and $B$ are now at potentials $V_A$ and $V_B$, respectively
( $V_A - V_B = V$, the voltage of the battery).
The situation is not static any more, but charges move inside the conductor so that there is a net charge flow in a particular direction. We define this net flow of electric charge as "electric current."



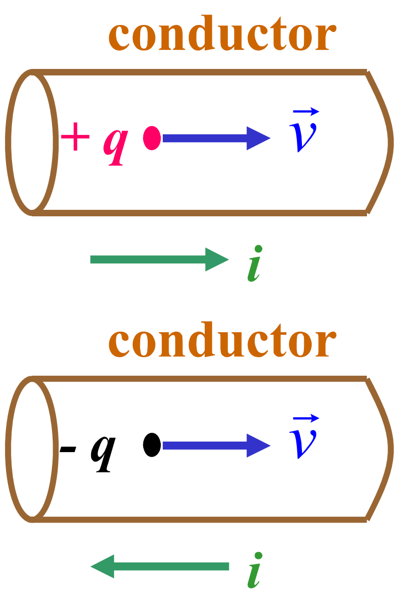

$$\boxed{i = \frac{dq}{dt}}$$

Consider the conductor shown in the figure. It is connected to a battery (not shown) and thus charges move through the conductor. Consider one of the cross sections through the conductor ( $aa'$ or $bb'$ or $cc'$ ).

The electric current $i$ is defined as $i = \frac{dq}{dt}$.

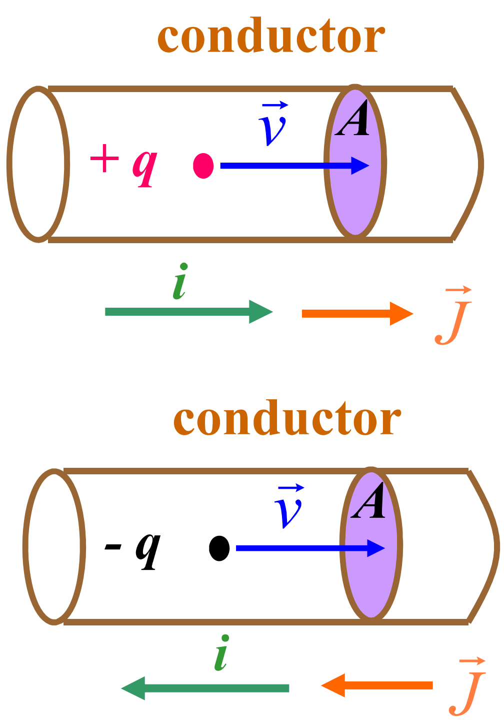
Current = rate at which charge flows

Current SI Unit: C/s, known as the "ampere"

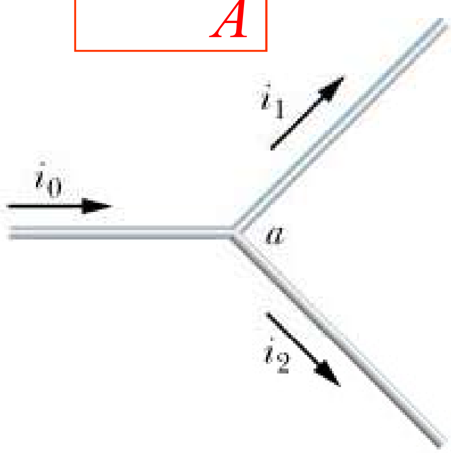



**Current Direction.** An electric current is represented by an arrow, which has the same direction as the charge velocity. For historical reasons we use the following convention:

A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.




$$\boxed{J = \frac{i}{A}}$$




## Current Density

Current density is a vector that is defined as follows:

Its magnitude is $J = \frac{i}{A}$ **SI unit for $J$: A / m$^2$**

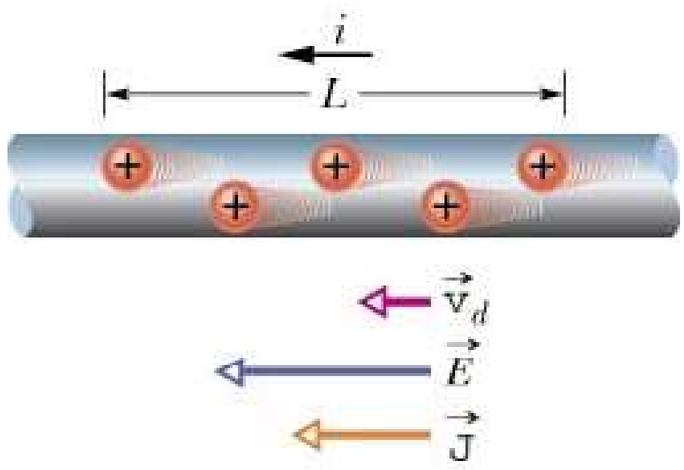
The direction of $\vec{J}$ is the same as that of the current.

The current through a conductor of cross-sectional area $A$ is given by the equation $i = JA$ if the current density is constant.

If $\vec{J}$ is not constant, then $i = \int \vec{J} \cdot d\vec{A}$.

We note that even though the current density is a vector the electric current is not. This is illustrated in the figure to the left. An incoming current $i_0$ branches at point $a$ into two currents, $i_1$ and $i_2$.

Current $i_0 = i_1 + i_2$. This equation expresses the conservation of charge at point $a$. Please note that we have not used vector addition.




## Drift Speed

When a current flows through a conductor the electric field causes the charges to move with a constant drift speed $v_d$. This drift speed is superimposed on the random motion of the charges.

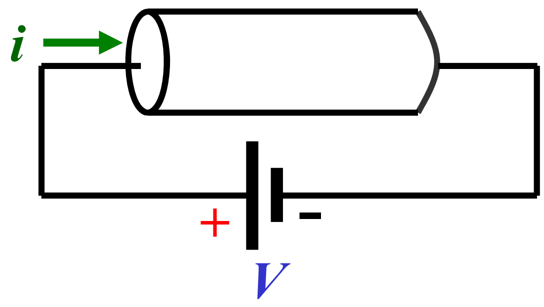
$\boxed{J = nv_d e}$ $\boxed{\vec{J} = ne\vec{v}_d}$

Consider the conductor of cross-sectional area $A$ shown in the figure. We assume that the current in the conductor consists of positive charges. The total charge $q$ within a length $L$ is given by $q = (nAL)e$, where n is a number of carriers per unit volume. This charge moves through area $A$

in a time $t = \dfrac{L}{v_d}$. The current is $i = \dfrac{q}{t} = \dfrac{nALe}{L/v_d} = nAv_d e.$

The current density is $J = \dfrac{i}{A} = \dfrac{nAv_d e}{A} = nv_d e.$

In vector form: $\vec{J} = ne\vec{v}_d$.




$$R = \frac{V}{i}$$


## Resistance

If we apply a voltage $V$ across a conductor (see figure) a current $i$ will flow through the conductor.

We define the conductor resistance as the ratio $R = \frac{V}{i}$ .

**SI Unit for $R$ :** $\frac{V}{A} =$ the ohm (symbol $\Omega$)

A conductor across which we apply a voltage $V = 1$ volt and results in a current $i = 1$ ampere is defined as having resistance of 1 $\Omega$.

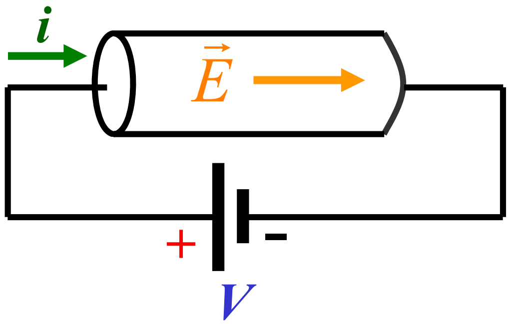
**Q :** Why not use the symbol "O" instead of "$\Omega$"?

**A :** Suppose we had a 1000 $\Omega$ resistor.

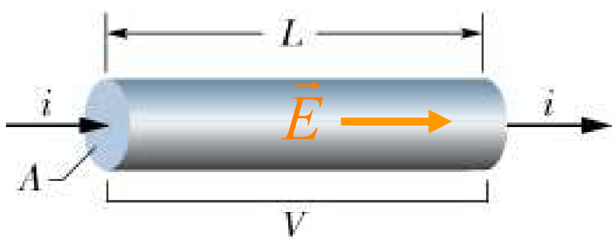
We would then write: 1000 O, which can easily be mistakenly read as 10000 $\Omega$.

A conductor whose function is to provide a specified resistance is known as a "resistor."

The symbol is given to the left.




$\boxed{\vec{E} = \rho \vec{J}}$ $\boxed{\vec{J} = \sigma \vec{E}}$




$\boxed{R = \rho \frac{L}{A}}$

**Resistivity**

Unlike the electrostatic case, the electric field in the conductor of the figure is not zero. We define as resistivity $\rho$ of the conductor the ratio $\rho = \frac{E}{J}$.

In vector form: $\vec{E} = \rho \vec{J}$.

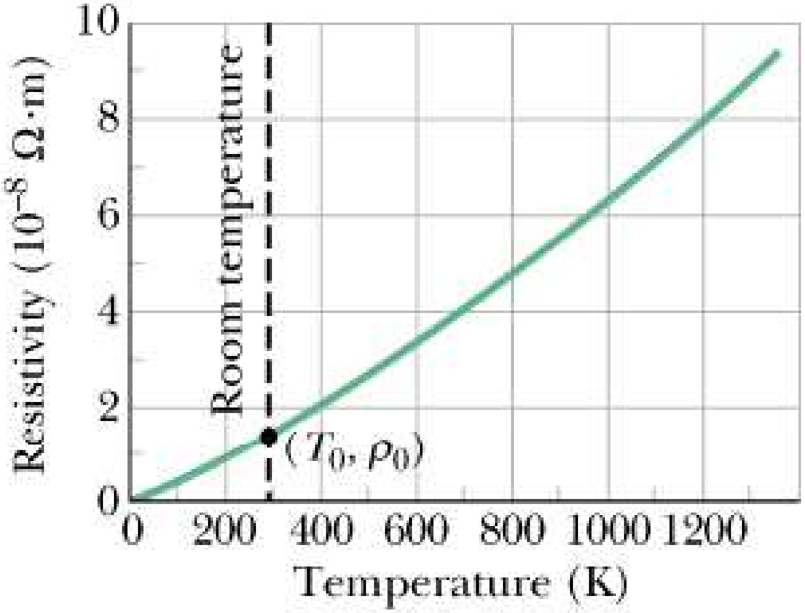
**SI unit for $\rho$:** $\frac{\text{V/m}}{\text{A/m}^2} = \frac{\text{V}}{\text{A}}\text{m} = \Omega \cdot \text{m}$

The conductivity $\sigma$ is defined as $\sigma = \frac{1}{\rho}$.

Using $\rho$, the previous equation takes the form: $\vec{J} = \sigma \vec{E}$.

Consider the conductor shown in the figure above. The electric field inside the conductor is $E = \frac{V}{L}$. The current density is $J = \frac{i}{A}$. We substitute $E$ and $J$ into equation $\rho = \frac{E}{J}$ and get: $\rho = \frac{V/L}{i/A} = \frac{V}{i}\frac{A}{L} = R\frac{A}{L} \rightarrow R = \rho \frac{L}{A}$.

$$\boxed{\rho - \rho_0 = \rho_0 \alpha (T - T_0)}$$



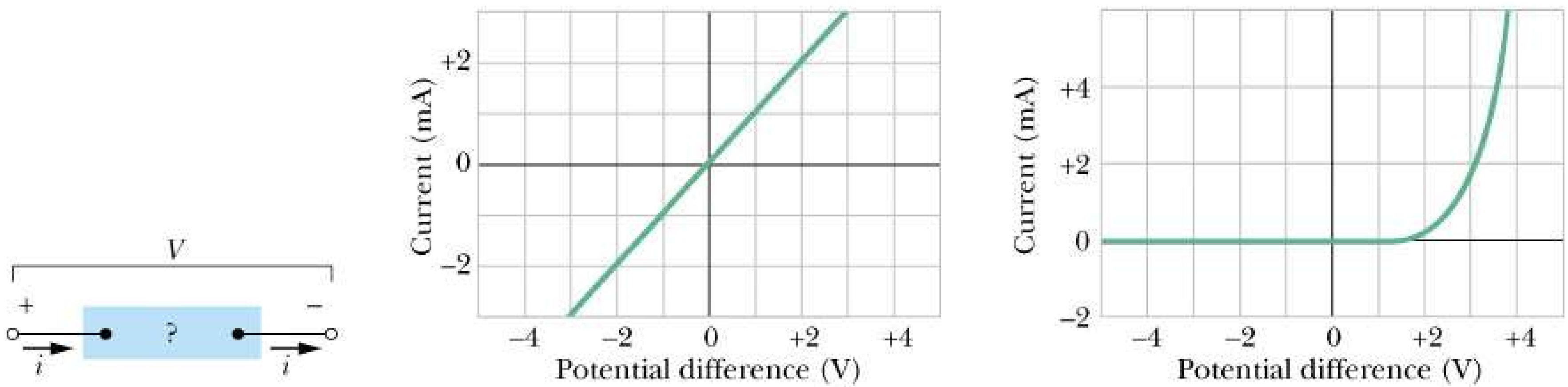

## Variation of Resistivity with Temperature

In the figure we plot the resistivity $\rho$ of copper as a function of temperature $T$. The dependence of $\rho$ on $T$ is almost linear. Similar dependence is observed in many conductors.

The following empirical equation is used for many practical applications:

$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$. The constant $\alpha$ is known as the "temperature coefficient of resistivity." The constant $T_0$ is a reference temperature usually taken to be room temperature ($T_0 = 293$ K ), and $\rho_0$ is the resistivity at $T_0$. For copper, $\rho_o = 1.69 \times 10^{-8}\ \Omega \cdot \text{m}$.

**Note:** Temperature enters the equation above as a difference $(T - T_0)$. Thus either the Celsius or the Kelvin temperature scale can be used.




(a) (b) (c)

**Ohm's Law.** A resistor was defined as a conductor whose resistance does not change with the voltage $V$ applied across it. In fig. $b$ we plot the current $i$ through a resistor as a function of $V$. The plot (known as the "$i$ - $V$ curve") is a straight line that passes through the origin. Such a conductor is said to be "**Ohmic**" and it obeys Ohm's law, which states: The current $i$ through a conductor is proportional to the voltage $V$ applied across it. Not all conductors obey Ohm's law (these are known as "**non - Ohmic**"). An example is given in fig. $c$ where we plot $i$ versus $V$ for a semiconductor diode. The ratio $V / i$ (and thus the resistance $R$ ) is not constant. As a matter of fact, the diode does not conduct for negative voltage values.

**Note :** Ohm's "law" is in reality a definition of Ohmic conductors (defined as the conductors that obey Ohm's law).